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EQUILIBRIUM OF A SYSTEM OF CRACKS WITH CONTACT AND OPENING REGIONS*

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The equilibrium of a system of rectilinear cracks is considered within the framework of the plane theory of elasticity, taking into account the possibility of the formation of contact regions on their surfaces. In this case a jump in the normal displacement is specified on a part of the crack surface (within the area of contact), and a normal stress in the opening region. The shear stress is specified along the whole crack.

The well-known integral equations (IE) obtained for cracks without contact regions /1-3/ cannot be used to solve the problem in question, since the loads in them are assumed given along the whole crack, whereas within the regions of contact between its surfaces the normal stresses are not known.

In order to overcome this difficulty, a different method of deriving the IE is proposed, describing the distribution of the jump in displacement along the crack. The possibility of representing the solution of the initial problem in the form of the sum of solutions of two problems for the initial crack, namely, of the problem of a crack with an unknown jump in shear displacements, but with shearing loads specified along it, and of the problem of determining the opening regions along the initial crack with unknown normal displacement jump and normal loads specified in these regions, is used. This makes it possible to obtain a system of IE written for the contact and opening regions, respectively, with separated right-hand sides. In one equation the right-hand side contains the known normal stresses, and in the other the shear stresses. The solution of the resulting system yields the distribution of displacement jumps along the crack and the unknown boundaries of the contact and opening regions. The condition determining the position of the opening regions is that there are no singularities in the stress distribution near the unknown boundaries of these regions /4/.

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The stresses within the areas of contact are calculated from their integral representations in terms of displacement jumps.

As an example, we consider the problem of a crack in a half-plane due to the action of shear and normal loads.

1. Formulation of the problem. Let us consider, in an elastic plane, using the XOY system of coordinates, a system of N rectilinear cuts L_k of length $2l_k$, along which regions of contact and opening may occur. The boundary conditions at the crack L_k have the following form /1/ in the local X_kOY_k system of coordinates (the Y_k axis is normal to the crack $|x_k| \leqslant l_k$):

$$\sigma_k = \sigma_k^0, \quad x_k \in D_k \subset L_k; \quad \tau_k = \tau_k^0, \quad x_k \in L_k$$
$$v^{(k)} = 0, \quad x_k \in F_k \subset L_k \tag{1.1}$$

F_k is the region of contact of the surfaces of the k-th crack, while D_k is the region of opening of the k-th crack, $v^{(k)} = v^{+(k)} - v^{-(k)}$ is the jump in the normal component of the displacement, $u^{(k)} = u^{+(k)} - u^{-(k)}$ is the jump in the shear component of the displacement, and σ_k^0, τ_k^0 are prescribed loads at the crack. We shall assume that there are no loads outside the crack. The passage to boundary conditions at the crack is made using the well-known Buckner method, and we used in /4/ for problems with contact regions.

In order to describe the stress-deformation state of an elastic body with cracks under boundary conditions (1.1), we shall use the integral representations connecting the jumps in displacements along the crack, with the stresses at the crack. These representations are mentioned e.g. when constructing a system of IE for the cleavage cracks, and have the form /1/

$$\pi P_n(x) = \int_{-l_n}^{l_n} \frac{g_n'\,dt}{t-x} + \sum_{\substack{k=1\\k\neq n}}^{N} \Big(\int_{-l_k}^{l_k} g_k'(t) K_{nk}(t,x) + \overline{g_k'(t)} L_{nk}(t,x) \Big) dt \tag{1.2}$$

$$\int_{-l_n}^{l_n} g_n'(t)\,dt = 0, \quad |x| \leqslant l_n, \quad n = 1,\ldots,N \tag{1.3}$$

$$P_n(x) = \sigma_n^0 - i\tau_n^0, \quad g_k'(x) = v_k'(x) - u_k'(x)$$
$$u_k(x) = \frac{2\mu}{(\varkappa+1)} u^{(k)}(x), \quad v_k(x) = \frac{2\mu}{(\varkappa+1)} v^{(k)}(x)$$

Here μ is the shear modulus, $\varkappa = 3 - 4\nu$ for plane deformation and $\varkappa = (3-\nu)/(1+\nu)$ for plane state of stress, ν is Poisson's ratio and $u^{(k)}(x)$, $v^{(k)}(x)$ are the displacement components at the k-th crack in a local X_kY_k system of coordinates. This expression was obtained under the condition that there are no displacement jumps at the ends of the crack (1.3). The kernels L_{nk} and K_{nk} are regular and their form can be found for a system of N cracks in a plane, in /1/. We note that the representations (1.2) for N cracks also retain their form and structure in various problems of the equilibrium of N cracks not only in a plane, but also in bodies of other shapes, the expression for the regular kernels K_{nk} and L_{nk} /1/ being the only ones that change. The expression for these kernels in the case of the loading N cracks in a half-plane, strip and a disc, can be found in /1/.

We will now consider a useful feature of representations (1.2) with condition (1.3), which can be used to calculate the stresses along any straight line outside N cracks in bodies of different shapes for which the kernels K_{nk}, L_{nk} are known. This is usually carried out using the well-known Kolosov-Muskhelishvili representation, which is laborious. Thus, let N cracks exist in a body of prescribed geometry and let the kernels K_{nk}, L_{nk} (1.2) be known. We need to find the distributions of the stresses along a straight line with direction α, passing through the point (x^0, y^0). We place along this direction the $(N+1)$-th crack with displacement jump $g_{N+1} = 0$ whose centre lies at the point (x^0, y^0) and the length l_{N+1} is arbitrary. Then the relations (1.2) will yield the distribution of the stresses $P_{N+1}(x)$ along this direction in the local $X_{N+1}Y_{N+1}$ system of coordinates. The above procedure enables us to calculate the stresses without resorting to the Kolosov-Muskhelishvili formulas, by directly using the well-known expressions for the kernels K_{nk}, L_{nk}.

If on the other hand the boundary conditions are given in terms of the stresses along the whole crack (or system of cracks), then representation (1.2) will lead directly to integral equations that are singular with respect to derivatives of the displacement jump which are, in this case, unknown along the whole crack.

In the present case, when contact regions may be generated, the boundary conditions are given in the zones of the crack opening in terms of the stresses, and in the contact zones partly in displacements and partly in stresses. The unknown quantities are the displacement jumps in the opening regions, contact stress and the jump in displacement shear component in the contact zones, as well as the boundary of the contact zone. In order to arrive in this case at the integral equation for the unkown displacement jumps starting from (1.2), we propose below a method of transforming the initial problem leading to representations of the type (1.2), written for the unknown components of displacement jumps in terms of the known load components for the corresponding regions.

2. Derivation of the system of integral equations. We shall first assume that the boundaries of the opening zones are given and consider an approach connected with obtaining a representation analogous to (1.2), which will enable us to derive a system of integral equations describing the displacement jumps at the cracks with contact and opening regions corresponding to the boundary-value problem (1.1). After obtaining such a system, we shall supplement it with a method of determining the unknown boundaries of the contact and opening regions.

We shall number the opening regions along the k-th crack thus: $1_k, 2_k, \ldots, i_k$, to the left we have $(x_k = -l_k)$ and to the right we have $(x_k = l_k)$, in the local X_kY_k system of coordinates.Therefore, we have in the i_k opening regions $(i_k \geqslant 1)$ for the k-th crack (the case of $i_k = 0$ has no opening regions). Let us use relations (1.2) with condition (1.3). The relations determine the stresses at the crack depending on the displacement jumps. We shall assume, when constructing the system integral of equations, that the displacement jumps in the opening and contact regions are known. Then the stresses at the line of the crack governed by these displacement jumps can be found using formulas (1.2). Indeed, writing these relations separately for the normal and shear stresses and passing to the local coordinate systems associated with the centres of regions of shear (they are the cracks L_n, $n = 1, \ldots, N$) and with centres of the regions of contact, we obtain the stresses in these regions in terms of the corresponding displacement jumps.

Let the stresses $\sigma_n^{(A)}(x)$, $\tau_n^{(A)}(x)$, $x \in L_n$ $(n = 1, \ldots, N)$ correspond to the shear displacement jump along the crack L_n, and the stresses $\sigma_n^{(B)}(x)$, $\tau_n^{(B)}(x)$, $x \in L_n$ $(n = 1, \ldots, N)$ to the jump in the normal component of the displacements. The sum of these stresses in the corresponding regions must be equal to the applied loads

$$\sigma_n^{(A)}(x) + \sigma_n^{(B)}(x) = \sigma_n^{(0)}(x), \quad x \in D_n^{(j)} \quad (j = 1_n, \ldots, i_n, \; n = 1, \ldots, N)$$
$$\tau_n^{(A)}(x) + \tau_n^{(B)}(x) = \tau_n^{(0)}(x), \; x \in L_n \quad (n = 1, \ldots, N)$$

Thus we arrive at the system integral equations, written separately for the shear and normal components of the loads in the corresponding regions

$$\int_{-l_n}^{l_n} \frac{u_n^{\prime(0)}\,dt}{t-x} + \sum_{\substack{k=1\\k\neq n}}^{N} \int_{-l_k}^{l_k} u_k^{\prime(0)} \operatorname{Re}(K_{nk}^{(00)} - L_{nk}^{(00)})\,dt - \tag{2.1}$$
$$\sum_{k=1}^{N}\sum_{j=1}^{i_k} \int_{-l_k}^{l_k} v_k^{\prime(j)} \operatorname{Im}(K_{nk}^{(0j)} + L_{nk}^{(0j)})\,dt = \pi\tau_n^{0}, \quad |x| \leqslant l_n$$
$$\int_{-l_n^{(j)}}^{l_n^{(j)}} \frac{v_n^{\prime(j)}\,dt}{t-x} + \sum_{k=1}^{N} \int_{-l_k}^{l_k} u_k^{\prime(0)} \operatorname{Im}(K_{nk}^{(j0)} - L_{nk}^{(j0)})\,dt +$$
$$\sum_{k=1}^{N}\sum_{f=1}^{i_k} \int_{-l_k}^{l_k} v_k^{\prime(f)} \operatorname{Re}(K_{nk}^{(jf)} + L_{nk}^{(jf)})\,dt = \pi\sigma_n^{0(j)}, \; |x| \leqslant l_n^{(j)}$$
$$\int_{-l_n}^{l_n} u_n^{\prime(0)}\,dt = 0, \qquad \int_{-l_n^{(j)}}^{l_n^{(j)}} v_n^{\prime(j)}\,dt = 0, \quad 1 \leqslant n \leqslant N, \quad 1_n \leqslant j \leqslant i_n$$

where $l_n^{(j)}$ is the length of the opening regions and $(x_n^{0(j)}, y_n^{0(j)})$ are the coordinates of the centres of the opening regions $(j = 1_n, \ldots, i_n;\; n = 1, \ldots, N)$.

If there are no contact regions $l_k^{(j)} = l_k$ on some crack, then system (2.1) will be identical with (1.2) obtained in /1/, but written separately for different stresses. We note that the opening regions are always denoted by a double index. The subscript refers to the number of the crack, and the superscript to the opening region. The region of shear has zero superscript. Therefore, when using the expression for the kernels K_{nk}, L_{nk} from /1/, we must replace in the case of the opening region, the corresponding index n or k by a double index $(\ldots_n{}^j$ or $\ldots_n{}^f)$, for example instead of K_{nk} we shall have $K_{nk}^{(jf)}$, where n is the number of the crack, j is the number of the opening region on it, and the shear regions have zero $(\ldots{}_n{}^0$ or $\ldots{}_k{}^0)$.

The resulting system of integral equations can be interpreted as a system of equations describing the equilibrium of N prescribed cracks with a shear displacement jump and $\sum i_k$ in k from 1 to N cracks with a normal displacement jump, whose centres are situated in the opening regions $(x_k{}^{0(j)}, y_k{}^{0(j)})$ and the lengths are equal to $2l_k{}^{(j)}$.

Thus we obtain a system of equations describing the equilibrium systems of cracks with contact and opening regions, and on the left-hand side of the integral with the Cauchy kernel is taken over the region in which the loads appearing on the right-hand side are specified, and this enables us to solve this system of equations.

As we said before, relation (1.2) with condition (1.3) also retains its form in bodies of any shape, and only the form of kernels K_{nk}, L_{nk} changes and can be found in /1/ for a disc and a half-plane. Therefore the system of Eqs.(2.1) will also describe the equilibrium of N cracks with contact and opening regions in this case for the corresponding kernels K_{nk}, L_{nk}.

System (2.1) in dimensionless form is analogous to the usual system of singular integral equations for a system of cracks of length equal to two /1/. A numerical solution of such a system can be carried out either using the well-known method of mechanical quadratures /1/, or by regularizing and reducing it to a system of Fredholm integral equations /5/.

However, in the case when the contact regions appear on the cracks, the system (2.1) will contain unknown boundaries of the contact regions $1_n \leqslant j \leqslant i_n$, $n = 1, \ldots, N$, and we must therefore show how to determine them. According to the results of an analysis in /4/ the contact regions will appear at sites where the crack surfaces would otherwise overlap, were the contact regions not introduced. Therefore we shall take, as the opening region, to a first approximation, the region where no such overlap occurs, and extend it until the stress intensity factor at the boundary of the opening region becomes equal to zero /4/ to within prescribed accuracy.

3. ***Example.*** Let us consider a crack of length $2l$ whose centre lies at a depth H below the surface of the half-plane, directed at an angle α to this surface. We place the XY system of coordinates at the boundary of the half-plane (with the Y axis directed inwards through the centre of the crack).

We shall assume that only a single opening region forms at the cracks, and denote its length by $2a$. The kernels K_{nk}, L_{nk} are obtained for the system of Eqs.(1.2) in the case of opening cracks in /1/. Using their expression, we shall write the system of Eqs.(2.1) in the form

$$\int_{-a}^{a} \frac{v'\,dt}{t-x_1} + \int_{-a}^{a} v' \operatorname{Re}(K_{11}^{(11)} + L_{11}^{(11)})\,dt + \int_{-l}^{l} u' \operatorname{Im}(K_{11}^{(10)} + L_{11}^{(10)})\,dt = \pi\sigma_1^{(1)0}, \quad |x_1| \leqslant a \tag{3.1}$$

$$\int_{-l}^{l} \frac{u'\,dt}{t-x_1} + \int_{-l}^{l} u' \operatorname{Re}(K_{1\cdot}^{(00)} - L_{11}^{(00)})\,dt - \int_{-a}^{a} v' \operatorname{Im}(K_{11}^{(01)} + L_{11}^{(01)})\,dt = \pi\tau_1^{(0)}, \quad |x_1| \leqslant l$$

$$K_{11}^{(jf)} = (1-\delta_{11}^{(jf)})\ \frac{e^{i\alpha}}{2}\left(\frac{1}{T_1^{(j)} - X_1^{(f)}} + \frac{e^{-2i\alpha}}{\bar{T}_1^{(j)} - \bar{X}_1^{(f)}}\right) +$$

$$\frac{e^{i\alpha}}{2}\left\{\frac{1}{X_1^{(j)} - \bar{T}_1^{(f)}} + \frac{e^{-2i\alpha}}{\bar{X}_1^{(j)} - T_1^{(f)}} + (\bar{T}_1^{(f)} - T_1^{(f)})\left[\frac{1+e^{-2i\alpha}}{(\bar{X}_1^{(j)} - T_1^{(f)})^2} - \frac{2e^{-2i\alpha}(X_1^{(j)} - T_1^{(f)})}{(\bar{X}_1^{(j)} - T_1^{(f)})^3}\right]\right\}$$

$$L_{11}^{(jf)} = (1-\delta_{11}^{(jf)})\ \frac{e^{-i\alpha}}{2}\left[\frac{1}{\bar{T}_1^{(f)} - X_1^{(j)}} + \frac{T_1^{(f)} - X_1^{(j)}}{(T_1^{(f)} - \bar{X}_1^{(j)})^2} e^{-2i\alpha}\right] +$$

$$\frac{e^{-i\alpha}}{2}\left[\frac{T_1^{(f)} - \bar{T}_1^{(f)}}{(X_1^{(j)} - \bar{T}_1^{(f)})^2} + \frac{1}{\bar{X}_1^{(j)} - T_1^{(f)}} - e^{-2i\alpha}\ \frac{X_1^{(j)} - T_1^{(f)}}{(X_1^{(j)} - T_1^{(f)})^2}\right]$$

$$i,\ j = 0, 1;\quad \delta_{11}^{(jf)} = 1,\quad j = f;\quad 0,\ j \neq f;\ \delta_{11}^{(jf)} = 0,\qquad X_1^{(0)} = x_1 e^{i\alpha} - iH$$

$$T_1{}^{(0)} = te^{i\alpha} - iH,\quad X_1{}^{(1)} = (x_1 \mp d)\,e^{i\alpha} - iH;\quad T_1{}^{(1)} = (t \mp d)\,e^{i\alpha} - iH$$

Here d is the distance between the centres of the crack and the opening region (the minus sign is used if the coordinate of the centre of the opening region in the local system of coordinates $X_1 0 Y_1$ is $x_1{}^{(1)0} > 0$, and plus if $x_1{}^{(1)0} < 0$).

System (3.1) is simplest in the case when the crack is parallel to the boundary of the half-plane $(\alpha = 0)$. In dimensionless form system (3.1) when $\alpha = 0$ will become,

$$\int_{-1}^{1} \frac{v'\,dt}{t-\xi} \int_{-1}^{1} v'\,dt\left[\frac{\xi - t}{D} - 4H_1{}^2(\xi - t)\left[\frac{(\xi - t)^2 - 12H_1{}^2}{D^3} + \frac{8H_1{}^2}{D^3}\right]\right] + \tag{3.2}$$

$$\int_{-1}^{1} \frac{u' 8H_2^3 ((a\xi/l - t \mp \Delta_1)^2 - 4H_2^2)\, dt}{D_1^3} = \pi\sigma_1^{(1)0}$$

$$\int_{-1}^{1} \frac{u'\, dt}{t-\xi} + \int_{-1}^{1} u'\, dt \left[\frac{\xi - t}{D_2} - 4H_2^2 \frac{(\xi - t)}{D_2^3} [(\xi - t)^2 - 12H_2^2] - \right.$$
$$\left. \int_{-1}^{1} v'\, dt \frac{8H_1^3}{D_3^3} (3\xi\, l/a - t \pm \Delta_2)^2 - 4H_1^2)\, dt = \pi\tau_1^0, \quad |\xi| \leqslant l$$

$$H_1 = H/a, \quad H_2 = H/l, \quad D = (\xi - t)^2 + 4H_1^2$$
$$D_1 = (a\xi/l - t \mp \Delta_1)^2 + 4H_2^2, \quad \Delta_1 = d/l$$
$$D_2 = (\xi - t)^2 + 4H_2^2$$
$$D_3 = (\xi l/a - t \pm \Delta_2)^2 + 4H_1^2, \quad \Delta_2 = d/a$$

When there is no region of contact, system (3.2) will be identical with one obtained earlier /1-3/.

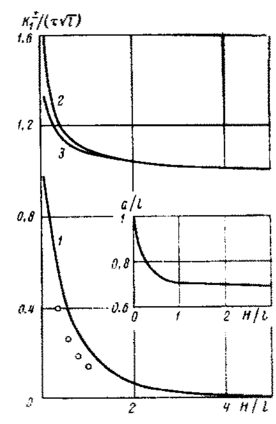
We see that the solution for a crack parallel to the boundary under the action of a shear stress $\tau_1^0 = -\tau, \sigma_1^{(1)0} = 0$ only, is antisymmetric with respect to the centre of the crack. Consequently K_1^+ (which corresponds to $x_1 = l$) and K_1^- (which corresponds to $x_1 = -l$) will have opposite signs. According to the results in /1-3/ $K_1^- < 0$, which corresponds to $v(x_1) < 0$ when $x_1 \geqslant -l$ and to overlap of the crack surfaces. Therefore, a contact region will appear near the end $x_1 = -l$ whose size $2a$ can be found using the algorithm given at the end of Sect.2, from the conditions that the solution contains no singularities near this boundary. Also, we should take the lower sign in the integrand in (3.2).

Let us first consider the asymptotic expression for system (3.2) as $H \to \infty$, $l = \text{const}$ under the load $\tau_1^0 = -\tau, \sigma_1^{(1)0} = 0$. To a first approximation in l/H the jump in the shear displacement is equal to $u(x_1) = \tau(1 - x_1^2/l^2)^{1/2}$. In this case the jump in normal displacement will be found from the second equation of (3.2) to a first approximation in l/H, whose solution has the form

$$v'(\xi) = \frac{3\pi\tau l^5}{256H^5} [4b^3(\xi^4 - {}^1/_2\xi^2 - {}^1/_8) + 12b^2\Delta_1(\xi^2 - {}^1/_2)\xi + 12b\Delta_1^2(\xi^2 - {}^1/_2) + \tag{3.3}$$
$$4\Delta_1^3\xi + 3b(\xi^2 - {}^1/_2) + 3\Delta_1\xi]/(1 - \xi^2)^{1/2}$$
$$b = a/l, \quad \Delta_1 = 1 - b, \quad \xi = x_1/a, \quad |x_1| \leqslant a, \quad |\xi| \leqslant 1$$

Determining the unknown boundary of the contact zone from the condition that there are no singularities when $\xi = -1$, we arrive at the equation $f(b) \equiv 35b^3 - 60b^2 + 45b - 14 = 0$. We have $f(0) = -14, f(\infty) > 0$, therefore there exists at least one real root. Confirming the fact that $f'(b) > 0, 0 \leqslant b \leqslant \infty$, we conclude that only one real root exists, which is equal to $b \simeq 0.69$, $a = 0.69l$.

Thus we have found that to a first approximation the opening region does not depend on the depth H, nor on the magnitude of shear stress τ. According to (3.3), when $H \to \infty$, the value of the opening of the crack tends to zero as l^5/H^5, at every point of the opening region, while the size of the opening region remains unchanged and equal to $a = 0.69l$. In the limit when $H = \infty$, there is no opening and the solution is identical with the solution of the problem of a shear crack in an infinite plane.



We note that an analogous result is obtained for a crack running along a circumference of radius R, and for a crack in the form of a strip on a cylindrical surface of radius R under a shear load /7, 8/. In this case, when the crack is under a shear stress, an opening region is also generated whose length does not depend on R as $R \to \infty$ and is finite, while the size of the opening of the crack surface tends to zero.

System (3.2) was solved numerically using the method of mechanical quadratures, for certain parameters $\tau_1^0 = -\tau, \sigma_1^{(1)0} = \sigma$ and H/l (we recall that we take the lower sign in (3.2)). We seek the length $2a$ of the opening region by consecutive approximations using the algorithm of Sect.2.

The figure shows the dependence on H/l of the stress intensity factors for $\sigma = 0$: $K_1^+/(\tau\sqrt{l})$ (curve 1), $K_2/(\tau\sqrt{l})$ (curve 2) and $K_2^-/(\tau\sqrt{l})$ (curve 3). The plus sign corresponds to the tip of the crack $x_1 = l$ and the minus sign to the tip of the crack $x_1 = -l$. The dots denote the results of computing $K_1^+/(\tau\sqrt{l})$ without taking into account the contact region. All these relations imply that the stress intensity factor K_1^+ becomes larger than the analogous factor calculated without taking into account the contact region. The change in the length $2a$ of the opening region is also shown, depending in this case on H/l.

The ratio σ/τ for which the crack is either fully open (σ_t) or fully closed (σ_c): $\sigma_t/\tau =$ 0.13, 0.088, 0.002, $\sigma_c/\tau =$ −0.29, −0.19, −0.005, was calculated for $H/l = 0.6, 1, 5$.

The change as a function of σ/τ in the length $2a$ of the opening region and in the stress intensity factors for $H/l = 0.6$, were also calculated:

σ/τ	−0.29	−0.20	−0.12	0.00	0.06	0.13
a/l	0.00	0.42	0.55	0.73	0.83	1.048
$K_2^+/(\tau\sqrt{l})$	1.111	1.112	1.116	1.139	1.148	1.177
$K_2^-/(\tau\sqrt{l})$	1.111	1.109	1.060	1.090	1.075	1.048
$K_1^+/(\tau\sqrt{l})$	0.000	0.116	0.207	0.506	0.444	0.534

The above results show that when $\sigma \leqslant \sigma_c < 0$, we have $K_1^\pm = 0$, and for $\sigma_c < \sigma < \sigma_t$ we have $K_1^- = 0$ and the limiting equilibrium of the crack will be determined, in this case, by the stress intensity factor $K_2^\pm$ for $\sigma \leqslant \sigma_c < 0$ and $K_2^\pm$, K_1^+ for $\sigma_t < \sigma$.

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